Antiderivative

№1. Find out if the the function $F(x)$ is an antiderivative of a function $f(x)$:

1. $F(x) = 3x^2 + x - 2, \quad f(x) = 6x + 1;$
2. $F(x) = \cos x + 3, \; f(x) = -\sin x + 3;$
3. $F(x) = \sqrt{2x+1}, \; f(x) = \dfrac{1}{\sqrt{2x+1}}.$

№2. Show that

1. $\displaystyle\int x\cos x \, \mathrm{d}x = \cos x + x\sin x + C;$
2. $\displaystyle\int \frac{x}{\sqrt{x^2+4}} \, \mathrm{d}x = \sqrt{x^2+4} + C.$

№3. For $f(x)$ find its antiderivative which passes through the specified point:

1. $f(x) = x^2, \quad A(-1;3);$
2. $f(x) = \sin x, \quad B(\pi;-1);$
3. $f(x) = e^x, \quad C(0;-6);$
4. $f(x) = \dfrac{1}{x^2}, \quad D\left(\dfrac{1}{3};-9\right);$
5. $f(x) = \dfrac{1}{\cos^2 x}, \quad E\left(\dfrac{\pi}{3};3\sqrt{3}\right);$
6. $f(x) = \dfrac{1}{x}, \quad F(e^{-3};7);$
7. $f(x) = \dfrac{1}{x^4}, \quad G\left(-\dfrac{1}{2};3\right).$

№4. For the function $f(x) = \sin^2\dfrac{x}{2} - \cos^2\dfrac{x}{2}$ find any two antiderivatives such that the distance between the corresponding points (that is, points with the same abscissa) is equal to 2.

№5. Find the general form of the antiderivatives of the function

$$f(x) = \begin{cases} 2x, & \text{if } x < 0, \\ \sin x, & \text{if } x \geq 0. \end{cases}$$

№6. An odd function defined on $\mathbb{R}$ has an antiderivative. Prove that this antiderivative is an even function.