

Definite integral

№1. For the function f find an antiderivative F whose graph passes through the given point:

1. $f(x) = \frac{1}{2} \cos \frac{x}{2} - 5 \sin 5x, \quad A(\pi, 0);$
2. $f(x) = \frac{6}{\cos^2\left(6x + \frac{\pi}{12}\right)}, \quad A\left(\frac{\pi}{24}, \frac{\sqrt{3}}{2}\right);$
3. $f(x) = 5^x \ln 5 + 2^x \ln 2, \quad B(2, -3);$
4. $f(x) = \frac{1}{\sqrt{7x-5}}, \quad A(3, 2);$
5. $f(x) = \sin x - 3e^x, \quad B(0, 5);$
6. $f(x) = 16x^3 - e^{x/2}, \quad B(1, -2\sqrt{e}).$

№2. (*Ireland, 2025*) $k \in \mathbb{R}$ is a constant, and

$$\int_0^k e^{5x} \, dx = 9.$$

Use this to find the value of k .

№3. Calculate the following integral:

$$\int_0^{2025} \{x\} \, dx.$$

Here $\{a\}$ denotes a fractional part of a .