# Logarithm of a number

$$a^b = c \iff b = \log_a c$$

*Remark:* There are several notations for some common logarithms:

$$\log_{10} a = \lg a, \qquad \log_e a = \ln a.$$

Here $e$ stands for Euler's number, also called a natural constant, for this reason $\ln a$ is usually called a *natural logarithm*. We'll talk about this remarkable number later, now you can just accept it or google it :-)

**№1.** Calculate:

1. $\log_2 8$;
2. $\log_{13} \frac{1}{13}$;
3. $\log_{14} 1$;
4. $\log_{20} 20$;
5. $\log_5 0.04$;
6. $\log_{81} 3$;
7. $\lg 0.001$;
8. $\log_{36} 216$;
9. $\log_{0.2} 5$.

**№2.** Prove the following properties of a logarithm:

1. $a^{\log_a b} = b$;
2. $\log_a b^k = k \cdot \log_a b$;
3. $\log_a^n b = \frac{1}{n} \cdot \log_a b$;
4. $\log_a b + \log_a c = \log_a bc$;
5. $\log_a b - \log_a c = \log_a \frac{b}{c}$;
6. $\dfrac{\log_a b}{\log_a c} = \log_c b$;
7. $\log_a b = \dfrac{1}{\log_b a}$.

*Remark:* You should memorize all those properties.

**№3.** Find the value of the expression:

1. $\log_{\frac{1}{3}}(\log_2 512)$;
2. $\log_2 32 - \log_{21} \sqrt{21} - 3\log_4 \frac{1}{64}$;
3. $\log_{18} 36 + \log_{18} 9$;
4. $\log_{13} 26 - \log_{13} 2$;
5. $\log_{64} \sqrt[3]{2}$;
6. $\dfrac{\lg 27}{\lg 3}$;
7. $10^{2\lg 7}$;
8. $27^{1-\log_3 4}$;
9. $5^{\frac{4}{\log_3 5}}$.

**№4.** Solve the equations:

1. $3^x = 5$;
2. $7^{2x-3} = 6$;
3. $2^{x+9} = 12$.

**№5.** Compute the value of the expression:

$$3^{\frac{2}{\log_{\sqrt{5}} 3} + \frac{1}{3}\log_3 8} - 27 \log_2 \sqrt[4]{2\sqrt[3]{2}}.$$

**№6.** Express $\log_{35} 28$ in terms of $a$ and $b$, if $a = \log_{14} 7$ and $b = \log_{14} 140$.

**№7.** Elements of a geometric progression are positive numbers. Prove that logarithms of consecutive elements of the progression with any base make an arithmetic progression.

**№8.** Calculate $\dfrac{1}{\log_2 20!} + \dfrac{1}{\log_3 20!} + ... + \dfrac{1}{\log_{20} 20!}$