Sum of an infinite geometric progression

№1. Compute the sum

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots$$

Hint: Imagine that you are eating a cake piece by piece :-)

If $|q|<1$, the terms of a geometric progression decrease, and we can compute the sum of infinitely many terms of such a sequence:

$$S_\infty=\frac{b_1}{1-q}.$$

№2. Compute the sum of the infinite geometric progression with first term $b_1=24$ and common ratio $q=\frac{3}{4}$.

№3. Prove that the number $1+3+3^2+\dots+3^{6n-1}$ is divisible by 364 for any $n\in\mathbb{N}$.

№4. Calculate the sum $1+2\cdot 5+3\cdot 5^2+\dots+100\cdot 5^{99}$.